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STRESS–STRAIN STATE OF A COMPOSITE ANISOTROPIC PLATE WITH CURVILINEAR CRACKS AND THIN RIGID INCLUSIONS

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A complex-potential solution of a mixed problem of the linear theory of elasticity is given for an infinite plate composed of two anisotropic half-planes. The plate contains cuts and thin undeformable inclusions shaped like arbitrary open smooth curves that do not intersect each other and the interface between the half-planes.

Formulations of the Problem. We consider a piecewise homogeneous plate occupying the plane $z = x+iy$. The plate is composed of two anisotropic half-planes bonded continuously along the line $x = 0$. At infinity, stresses are specified for which the continuity conditions

$$\sigma_x^{(1)} = \sigma_x^{(2)}, \quad \tau_{xy}^{(1)} = \tau_{xy}^{(2)}, \quad u_y'^{(1)} = u_y'^{(2)}, \quad v_y'^{(1)} = v_y'^{(2)} \tag{1}$$

are satisfied on the interface of the half-planes if the plate have no defects and stiffeners.

It is assumed that through cuts (cracks) and thin undeformable inclusions are located in the half-plane $x > 0$ along smooth curves $L_j = (a_j, b_j)$ for $j = 1, \ldots, k_1$ and for $j = k_1 + 1, \ldots, k$, respectively: $L = \bigcup\limits_{j=1}^{k_1} L_j$ and $C = \bigcup\limits_{j=k_1+1}^{k} L_j$. The curves do not intersect each other and the interface between the half-planes. For each curve, we choose the normals $\boldsymbol{n}(t)$ $(t \in L_j)$ directed to the right when moving from a_j to b_j. It is assumed that the edges of the cuts do not contact and they are subjected to the self-equilibrated, uniformly distributed loads

$$X_n^{\pm}(t) + iY_n^{\pm}(t) = \pm P(t), \quad t \in L. \tag{2}$$

The curvilinear inclusions can displace as a single rigid body:

$$u^{\pm}(t) + iv^{\pm}(t) = g_1(t) + ig_2(t) = G(t), \quad t \in C; \qquad G(t) = c_j + i\varepsilon_j t, \quad t \in L_j. \tag{3}$$

Here c_j is a complex constant and ε_j is the unknown or specified angle of rotation of the rigid inclusion L_j. The superscripts plus and minus correspond to the left and right edges of the cut or inclusion, respectively.

It is required to determine the stress–strain state of the plate. To solve the problem, we use the methods [1–5] for calculating the stresses and strains in anisotropic plates with defects and stiffeners of arbitrary shapes. Lin'kov [6] considered a similar problem and derived a system of integral equations for a plate composed of two isotropic half-planes.

Expressions for Potentials. Let $\mu_1^{(r)}$ and $\mu_2^{(r)}$ be the unequal roots of the characteristic equation [7] $a_{11}^{(r)}\mu^4 - 2a_{16}^{(r)}\mu^3 + (2a_{12}^{(r)} + a_{66}^{(r)})\mu^2 - 2a_{26}^{(r)}\mu + a_{22}^{(r)} = 0$, where $a_{ij}^{(r)}$ are the coefficients of strains in Hooke's law ($r = 1$ corresponds to the half-plane $x > 0$ and $r = 2$ to the half-plane $x < 0$). We assume that $\operatorname{Im}\mu_1^{(r)} > 0$ and $\operatorname{Im}\mu_2^{(r)} > 0$.

By analogy with [3, 4], we seek the Lekhnitskii potentials [7] in the form

$$\Phi_\nu^{(r)}(z_\nu^{(r)}) = \Phi_{\nu 0}^{(r)} + \Phi_{\nu 1}^{(r)}(z_\nu^{(r)}) + \Phi_{\nu 2}^{(r)}(z_\nu^{(r)}), \qquad \nu = 1, 2, \quad r = 1, 2. \tag{4}$$

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Here $z_\nu^{(r)} = x + \mu_\nu^{(r)} y$, $\Phi_{\nu 0}^{(r)}$ are determined by the forces at infinity for the plane without defects and stiffeners,

$$\Phi_{\nu 1}^{(1)}(z_\nu^{(1)}) = \frac{1}{2\pi i} \int\limits_L \frac{\omega_\nu(\tau)}{\tau_\nu^{(1)} - z_\nu^{(1)}} \, d\tau_\nu^{(1)} - \frac{l_\nu^{(1)} s_\nu^{(1)} \overline{\omega_1(\tau)}}{\bar{\tau}_1^{(1)} - s_\nu^{(1)} z_\nu^{(1)}} \, d\bar{\tau}_1^{(1)} - \frac{n_\nu^{(1)} m_\nu^{(1)} \overline{\omega_2(\tau)}}{\bar{\tau}_2^{(1)} - m_\nu^{(1)} z_\nu^{(1)}} \, d\bar{\tau}_2^{(1)},$$

$$\Phi_{\nu 2}^{(1)}(z_\nu^{(1)}) = \frac{1}{2\pi i} \int\limits_C \frac{\mu_\nu(\tau)}{\tau_\nu^{(1)} - z_\nu^{(1)}} \, d\tau_\nu^{(1)} - \frac{l_\nu^{(1)} s_\nu^{(1)} \overline{\mu_1(\tau)}}{\bar{\tau}_1^{(1)} - s_\nu^{(1)} z_\nu^{(1)}} \, d\bar{\tau}_1^{(1)} - \frac{n_\nu^{(1)} m_\nu^{(1)} \overline{\mu_2(\tau)}}{\bar{\tau}_2^{(1)} - m_\nu^{(1)} z_\nu^{(1)}} \, d\bar{\tau}_2^{(1)},$$

$$\Phi_{\nu 1}^{(2)}(z_\nu^{(2)}) = \frac{1}{2\pi i} \int\limits_L \frac{l_\nu^{(2)} s_\nu^{(2)} \omega_1(\tau)}{\tau_1^{(1)} - s_\nu^{(2)} z_\nu^{(2)}} \, d\tau_1^{(1)} + \frac{n_\nu^{(2)} m_\nu^{(2)} \omega_2(\tau)}{\tau_2^{(1)} - m_\nu^{(2)} z_\nu^{(2)}} \, d\tau_2^{(1)}, \tag{5}$$

$$\Phi_{\nu 2}^{(2)}(z_\nu^{(2)}) = \frac{1}{2\pi i} \int\limits_C \frac{l_\nu^{(2)} s_\nu^{(2)} \mu_1(\tau)}{\tau_1^{(1)} - s_\nu^{(2)} z_\nu^{(2)}} \, d\tau_1^{(1)} + \frac{n_\nu^{(2)} m_\nu^{(2)} \mu_2(\tau)}{\tau_2^{(1)} - m_\nu^{(2)} z_\nu^{(2)}} \, d\tau_2^{(1)},$$

$$s_\nu^{(1)} = \frac{\bar{\mu}_1^{(1)}}{\mu_\nu^{(1)}}, \qquad m_\nu^{(1)} = \frac{\bar{\mu}_2^{(1)}}{\mu_\nu^{(1)}}, \qquad s_\nu^{(2)} = \frac{\mu_1^{(1)}}{\mu_\nu^{(2)}}, \qquad m_\nu^{(2)} = \frac{\mu_2^{(1)}}{\mu_\nu^{(2)}},$$

$$d\tau_\nu^{(1)} = (\mu_\nu^{(1)} \cos\varphi(\tau) - \sin\varphi(\tau)) \, ds = M_\nu^{(1)}(\tau) \, ds,$$

$\varphi(\tau)$ is the angle between the normal $\boldsymbol{n}(\tau)$ and the x axis, and ds is the differential of the arc length. The vectors $\{\bar{l}_1^{(1)}, \bar{l}_2^{(1)}, l_1^{(2)}, l_2^{(2)}\}$ and $\{\bar{n}_1^{(1)}, \bar{n}_2^{(1)}, n_1^{(2)}, n_2^{(2)}\}$ are determined by conditions (1) and satisfy the system $A\boldsymbol{X} = \boldsymbol{B}^{(\nu)}$ (for $\nu = 1$ and $\nu = 2$, respectively), where

$$A = \begin{pmatrix} -1 & -1 & 1 & 1 \\ -\bar{\mu}_1^{(1)} & -\bar{\mu}_2^{(1)} & \mu_1^{(2)} & \mu_2^{(2)} \\ -\bar{p}_1^{(1)} & -\bar{p}_2^{(1)} & p_1^{(2)} & p_2^{(2)} \\ -\bar{q}_1^{(1)} & -\bar{q}_2^{(1)} & q_1^{(2)} & q_2^{(2)} \end{pmatrix}, \qquad \boldsymbol{B}^{(\nu)} = \{1, \mu_\nu^{(1)}, p_\nu^{(1)}, q_\nu^{(1)}\},$$

$$p_\nu^{(r)} = a_{11}^{(r)} (\mu_\nu^{(r)})^2 - a_{16}^{(r)} \mu_\nu^{(r)} + a_{12}^{(r)}, \qquad q_\nu^{(r)} = a_{12}^{(r)} \mu_\nu^{(r)} + a_{22}^{(r)} / \mu_\nu^{(r)} - a_{26}^{(r)}, \qquad \nu = 1, 2.$$

Thus, the continuity conditions (1) at the interface between the half-planes are satisfied automatically for arbitrary boundary conditions (2) and (3) on the cuts and rigid inclusions.

System of Integral Equations of the Problem. Using representations (4) and the Sokhotskii–Plemelj formulas, from the boundary conditions (2) and (3) we obtain the system of integral equations for determining the desired densities $\omega_1(t)$, $\omega_2(t)$, $\mu_1(t)$, and $\mu_2(t)$ and the relations for $\omega_1(t)$ and $\omega_2(t)$ for the cuts $\mu_1(t)$ and $\mu_2(t)$ and rigid inclusions:

$$\begin{gathered} \int\limits_L \frac{\omega_1(\tau)}{\tau_1^{(1)} - t_1^{(1)}} \, d\tau_1^{(1)} + \int\limits_L \omega_1(\tau) K_{11}(t,\tau) \, ds + \int\limits_L \overline{\omega_1(\tau)} K_{12}(t,\tau) \, ds \\ + \int\limits_C \mu_1(\tau) K_{13}(t,\tau) \, ds + \int\limits_C \overline{\mu_1(\tau)} K_{14}(t,\tau) \, ds = f_1(t), \quad t \in L, \\ \int\limits_C \frac{\mu_1(\tau)}{\tau_1^{(1)} - t_1^{(1)}} \, d\tau_1^{(1)} + \int\limits_C \mu_1(\tau) K_{21}(t,\tau) \, ds + \int\limits_C \overline{\mu_1(\tau)} K_{22}(t,\tau) \, ds \\ + \int\limits_L \omega_1(\tau) K_{23}(t,\tau) \, ds + \int\limits_L \overline{\omega_1(\tau)} K_{24}(t,\tau) \, ds = f_2(t), \quad t \in C, \\ a(t)\omega_1(t) + b(t)\overline{\omega_1(t)} + \omega_2(t) = 0, \qquad t \in L, \\ A(t)\mu_1(t) + B(t)\overline{\mu_1(t)} + \mu_2(t) = 0, \qquad t \in C. \end{gathered} \tag{6}$$

Here

$$f_1(t) = \frac{\pi i \overline{F(t)}}{\overline{b(t)}} - \pi i \Big[\frac{\overline{a(t)}}{\overline{b(t)}}\,\bar{\Phi}_{10}^{(1)} + \Phi_{10}^{(1)} + \frac{1}{\overline{b(t)}}\,\bar{\Phi}_{20}^{(1)}\Big], \qquad t \in L;$$

$$f_2(t) = \frac{\pi i \overline{W(t)}}{\overline{B(t)}} - \pi i \Big[\frac{\overline{A(t)}}{\overline{B(t)}}\,\bar{\Phi}_{10}^{(1)} + \Phi_{10}^{(1)} + \frac{1}{\overline{B(t)}}\,\bar{\Phi}_{20}^{(1)}\Big], \qquad t \in C;$$

$$a(t) = a_0\,\frac{M_1^{(1)}(t)}{M_2^{(1)}(t)}; \qquad b(t) = b_0\,\frac{\overline{M_1^{(1)}(t)}}{M_2^{(1)}(t)}; \qquad a_0 = \frac{\mu_1^{(1)} - \bar{\mu}_2^{(1)}}{\mu_2^{(1)} - \bar{\mu}_2^{(1)}}; \qquad b_0 = \frac{\bar{\mu}_1^{(1)} - \bar{\mu}_2^{(1)}}{\mu_2^{(1)} - \bar{\mu}_2^{(1)}};$$

$$A(t) = A_0\,\frac{M_1^{(1)}(t)}{M_2^{(1)}(t)}; \quad B(t) = B_0\,\frac{\overline{M_1^{(1)}(t)}}{M_2^{(1)}(t)}; \quad A_0 = \frac{\bar{p}_2^{(1)} q_1^{(1)} - p_1^{(1)} \bar{q}_2^{(1)}}{\bar{p}_2^{(1)} q_2^{(1)} - p_2^{(1)} \bar{q}_2^{(1)}}; \quad B_0 = \frac{\bar{p}_2^{(1)} \bar{q}_1^{(1)} - \bar{p}_1^{(1)} \bar{q}_2^{(1)}}{\bar{p}_2^{(1)} q_2^{(1)} - p_2^{(1)} \bar{q}_2^{(1)}};$$

$$F^{\pm}(t) = F(t) = \pm\,\frac{X_n^{\pm}(t) + \bar{\mu}_2^{(1)} Y_n^{\pm}(t)}{(\mu_2^{(1)} - \bar{\mu}_2^{(1)}) M_2^{(1)}(t)}; \qquad W^{\pm}(t) = W(t) = \frac{\bar{p}_2^{(1)}\,dg_2/ds - \bar{q}_2^{(1)}\,dg_1/ds}{(\bar{p}_2^{(1)} q_2^{(1)} - p_2^{(1)} \bar{q}_2^{(1)}) M_2^{(1)}(t)}.$$

The system is supplemented by the equations

$$\int\limits_{L_j} \omega_1(\tau)\,d\tau_1^{(1)} = 0, \quad j = 1, \ldots, k_1, \qquad \int\limits_{L_j} \mu_1(\tau)\,d\tau_1^{(1)} = 0, \quad j = k_1 + 1, \ldots, k, \tag{7}$$

which require that the displacements be unique after passing the contour of each cut and the principal vector of forces acting on each rigid inclusion be equal to zero.

The angles of rotation of rigid inclusions under plate loading are determined by the condition of vanishing of the principal moment of forces acting on each inclusion. This condition has the form

$$2\mathrm{Re}\Bigg\{\int\limits_{L_j} \Big(\tau_1^{(1)} - \tau_2^{(1)} A_0 - \bar{\tau}_2^{(1)} \bar{B}_0\Big)\mu_1(\tau)\,d\tau_1^{(1)}\Bigg\} = 0, \qquad j = k_1 + 1, \ldots, k. \tag{8}$$

Thus, we have obtained system (6)–(8) for determining the densities $\omega_1(t)$, $\omega_2(t)$, $\mu_1(t)$, and $\mu_2(t)$.

Using representations (5) and system (6)–(8) and calculating the limiting values of the anisotropy parameters as done in [1], one can obtain potentials and a system of equations for the cases where one or two half-planes are isotropic.

Numerical Solution. Introducing the parametrization of the curves $L_j = \{t = \tau^j(\xi),\ \xi \in [-1, 1]\}$ and the notation $\omega_1(\tau^j(\xi)) = \chi_j(\xi) = \chi_j^0(\xi)/\sqrt{1-\xi^2}$ $(j = 1, \ldots, k_1)$ and $\mu_1(\tau^j(\xi)) = \chi_j(\xi) = \chi_j^0(\xi)/\sqrt{1-\xi^2}$ $(j = k_1 + 1, \ldots, k)$, we reduce system (6)–(8) to the canonical system of integral equations

$$\sum_{p=1}^{k} \int\limits_{-1}^{1} \Big\{K_1^{jp}(\xi, \eta)\chi_p(\eta) + K_2^{jp}(\xi, \eta)\overline{\chi_p(\eta)}\Big\}\,d\eta = f_j(\xi), \qquad j = 1, \ldots, k,$$

$$\int\limits_{-1}^{1} \chi_j(\eta)(\tau_1^j(\eta))'\,d\eta = 0, \quad j = 1, \ldots, k, \qquad \mathrm{Re}\Bigg\{\int\limits_{-1}^{1} K^j(\eta)\chi_j(\eta)\,d\eta\Bigg\} = 0, \quad j = k_1 + 1, \ldots, k,$$

where the functions $K_1^{jj}(\xi, \eta)$ have the Cauchy-type singularities.

The system is solved with the use of quadrature formulas according to the scheme described in [3]. Once the solution is obtained, the potentials and stresses can be determined at each point of the plate with a specified accuracy [7] and the stress-intensity factors K_1 and K_2 at the tips of the cracks and rigid inclusions can be calculated [3]:

$$(\sigma_x^{(r)}, \tau_{xy}^{(r)}, \sigma_y^{(r)}) = 2\mathrm{Re}\Bigg\{\sum_{\nu=1}^{2} ((\mu_\nu^{(r)})^2, -\mu_\nu^{(r)}, 1)\Phi_\nu^{(r)}(z_\nu^{(r)})\Bigg\},$$

$$K_1 = \lim_{\substack{r \to 0 \\ \theta = 0}} \sigma_n \sqrt{2\pi r}, \qquad K_2 = \lim_{\substack{r \to 0 \\ \theta = 0}} \tau_n \sqrt{2\pi r}.$$

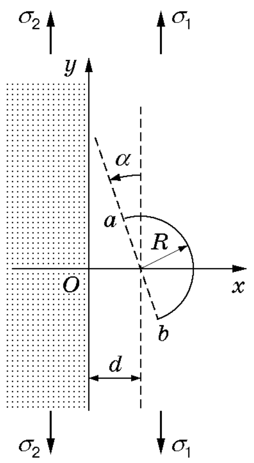

Fig. 1

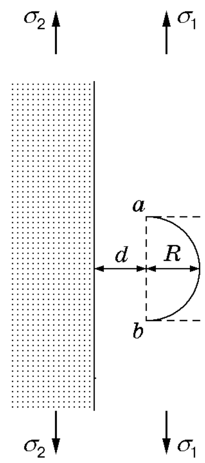

Fig. 2

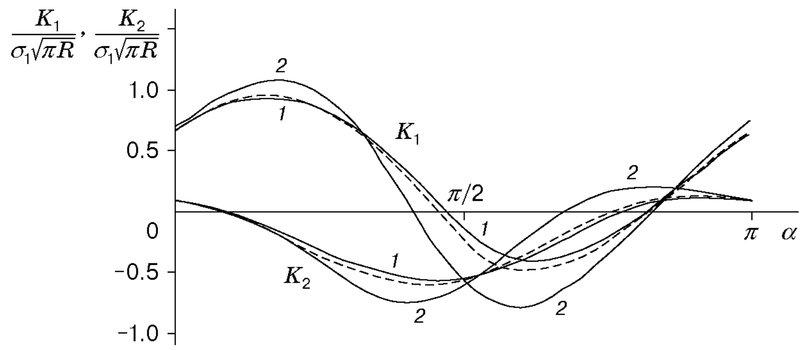

Fig. 3

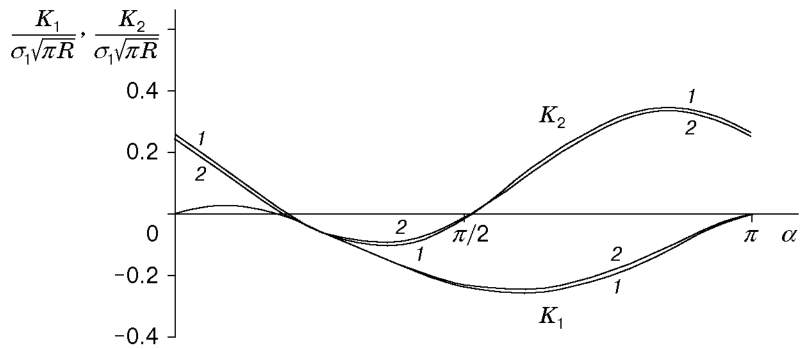

Fig. 4

Here r and θ are polar coordinates of the point (the pole is located at the tip of the curve, and the polar axis is tangent to it), $\sigma_n = 0.5(\sigma_x + \sigma_y) + 0.5(\sigma_x - \sigma_y)\cos 2\varphi + \tau_{xy}\sin 2\varphi$, and $\tau_n = -0.5(\sigma_x - \sigma_y)\sin 2\varphi + \tau_{xy}\cos 2\varphi$ (φ is the angle between the normal to the curve at its tip and the x axis).

Calculation Results. Below, we consider plates composed of orthotropic materials with different anisotropic properties, plates composed of orthotropic and isotropic materials, and a semi-infinite plate with a free edge. In calculations, an isotropic material is modeled by introducing a "weak anisotropy" [$G = 0.4999E/(1+\nu)$]. In this case, the half-plane $x > 0$ always consists of an orthotropic material with the characteristics $E_1 = 53.84$ GPa, $E_1/E_2 = 3$, $G = 8.63$ GPa, and $\nu = 0.25$, the principal axis of anisotropy corresponding to the modulus $E_1^{(1)}$ coincides with the Ox axis. The plate is uniformly loaded along the interface between the half-planes by the tensile stresses σ_1 and $\sigma_2 = k\sigma_1$ applied at infinity (Figs. 1 and 2).

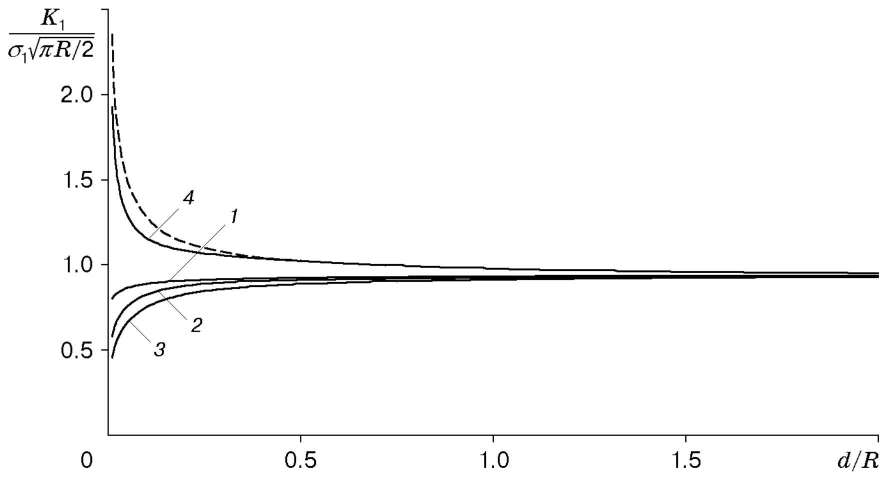

Fig. 5

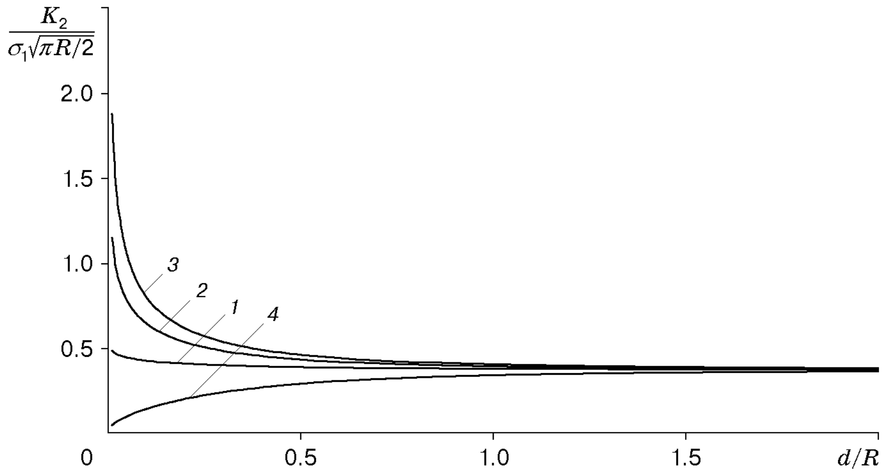

Fig. 6

Figures 3 and 4 show the stress-intensity factors at the tip a of the cut (Fig. 3) or a thin rigid inclusion (Fig. 4) shaped as a semicircumference versus the angle of rotation α (see Fig. 1). The ratio of the semicircumference radius to the distance to the interface is $R/d = 0.7$. The half-plane $x < 0$ is made of an orthotropic material with the characteristics $E_1 = 276.1$ GPa, $E_1/E_2 = 10$, $G = 10.35$ GPa, and $\nu = 0.25$ (curves 1) or it is absent (curves 2). In the first case, the principal axes of anisotropy that correspond to the moduli $E_1^{(1)}$ and $E_1^{(2)}$ are directed along the Ox axis and $k = E_2^{(2)}/E_2^{(1)}$. The problem of a cut in a half-plane is solved in [3]. The dashed curves in Fig. 3 refer to a homogeneous plate (plane) from a material of the right half-plane loaded by the stress σ_1 at infinity. For the rigid inclusion (Fig. 4), curves 1 and 2 are very close, and the curves corresponding to the homogeneous plate loaded by tensile stresses σ_1 coincide with curves 1 in Fig. 4.

Figures 5 and 6 show the stress-intensity factors at the tip a of the cut (Fig. 5) or rigid inclusion (Fig. 6) shaped as a semicircumference (solid curves) versus the ratio d/R [d is the distance from the tip to the interface between the half-planes (see Fig. 2) and R is the semicircumference radius]. The half-plane $x < 0$ is made of an orthotropic material with the characteristics $E_1 = 276.1$ GPa, $E_1/E_2 = 10$, $G = 10.35$ GPa, and $\nu = 0.25$ (curves 1 and 2 correspond to the cases where the axis of the modulus $E_1^{(2)}$ is directed along the Ox and Oy axes, respectively), an isotropic material with the characteristics $E_1 = 276.10$ GPa, $E_1/E_2 = 1$, and $\nu = 0.25$ (curves 3), or it is absent (curve 4). Curves 1–3 correspond to $k = E_2^{(2)}/E_2^{(1)}$, $E_1^{(2)}/E_2^{(1)}$, and $E^{(2)}/E_2^{(1)}$, respectively. For the case of the cut, the absolute values of the factors $K_2(a)$ are smaller than 0.15 for $d/R = 0.02$–2.0; the corresponding curves are not shown in Fig. 5. In Fig. 6, curves corresponding to the factor $K_1(a)$ are not shown (the absolute values of these factors are smaller than 0.04).

TABLE 1

k	d/a	$K_1(A)/(\sigma\sqrt{\pi a})$		$K_1(B)/(\sigma\sqrt{\pi a})$	
		Present results	Data of [8]	Present results	Data of [8]
0.5	1.04	1.0557	1.056	1.2497	1.250
	1.2	1.0396	1.040	1.1116	1.112
	2.0	1.0147	1.015	1.0250	1.025
	4.0	1.0040	1.004	1.0051	1.005
2.0	1.04	0.9547	0.955	0.8058	0.806
	1.2	0.9656	0.966	0.9031	0.903
	2.0	0.9863	0.986	0.9767	0.977
	4.0	0.9962	0.996	0.9951	0.995

It is of interest to compare these curves with dependences of the stress-intensity factors at the tip a of a rectilinear segment of length R on the ratio d/R in the problem of two rectilinear cuts or inclusions normal to the interface between the half-planes (see Fig. 2). The dependences of the factors $K_1(a)$ for the semicircumference and segment are very close for all composite plates considered. For the half-plane, these dependences differ substantially for $d < 0.5R$ [in Fig. 5, the dashed curve corresponding to the dependence of the factor $K_1(a)$ at the tip of the segment on the ratio d/R merges with curve 4 for large d/R]. In the case of rigid inclusions, the absolute values of the factors $K_2(a)$ at the segment tip are smaller than 0.04 for $d/R = 0.02$–2.0.

Table 1 summarizes the calculated stress-intensity factors for the problem of an internal crack AB of length $2a$ normal to the interface between two isotropic half-planes with different elastic properties, which are loaded along the interface by the tensile stresses σ_1 and $\sigma_2 = k\sigma_1$, $k = E^{(2)}/E^{(1)}$ ($\nu_1 = \nu_2 = 0.3$). The results obtained by the method proposed (20 interpolation points were used) are compared with the data of [8, Table 8.9] (B is the tip nearest to the interface).

The calculated stress-intensity factors at the tips of a rigid inclusion that approaches normally the edge of an isotropic half-plane coincide with those given in [9, Table 6] with an accuracy of 10^{-3}. It is noteworthy that there is a difference in determining these factors in the present work and in [9].

A comparison of the results obtained with the data available in the literature shows that the method proposed for calculating the stress–strain state of composite plates with cuts and thin undeformable inclusions is very effective and provides high accuracy.

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